Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Friday 7 June 2024

Morning (Time: 2 hours)

Paper reference **4PM1/02R**

Further Pure Mathematics
PAPER 2R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P74285RA

F:1/1/1/1/1/1/1/1/1/

P 7 4 2 8 5 R A 0 1 3 2

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The quadratic equation

$$kx^2 - (2k + 6)x + 16 = 0$$

has equal roots.

Find the two possible values of k

(4)

(Total for Question 1 is 4 marks)

2 (a) Expand $\frac{2}{\sqrt{1+3x}}$ in ascending powers of x up to and including the term in x^3

Express each coefficient as a fraction in its simplest terms where appropriate.

(4)

(b) State the range of values of x for which the expansion is valid.

(1)

Question 2 continued

(Total for Question 2 is 5 marks)

3 O, A and B are fixed points such that

$$\left|\overrightarrow{OA}\right| = 3\sqrt{5} \qquad \overrightarrow{AB} = \mathbf{i} + 3a\mathbf{j} \qquad \overrightarrow{OB} = 7\mathbf{i} + 2a\mathbf{j}$$

Given that $a > 0$

(a) find the value of a

(4)

(b) Hence find a unit vector parallel to $\overrightarrow{OA}$

(2)

Question 3 continued

(Total for Question 3 is 6 marks)

4 $f(x) = px^3 + qx^2 - 37x - 12q$ where p and q are constants.

When $f'(x)$ is divided by $(x + 2)$ the remainder is -33

Given that $(x + 5)$ is a factor of $f(x)$

(a) (i) show that $p = 2$

(ii) find the value of q

(6)

(b) Hence, use algebra to factorise $f(x)$ completely.

(3)

(c) Hence solve the equation $f(x) = 0$

(2)

Question 4 continued

(Total for Question 4 is 11 marks)

5 The force F newtons between two magnetic poles is given by the formula

$$F = \frac{3}{20r^2}$$

where r is the distance, in centimetres, between the poles.

The distance between the two poles is increasing at a constant rate of 0.7 cm/s

Find the rate of change of F, in newtons/s to 3 significant figures, when the distance between the poles is 2.8 cm

(6)

Question 5 continued

(Total for Question 5 is 6 marks)

6

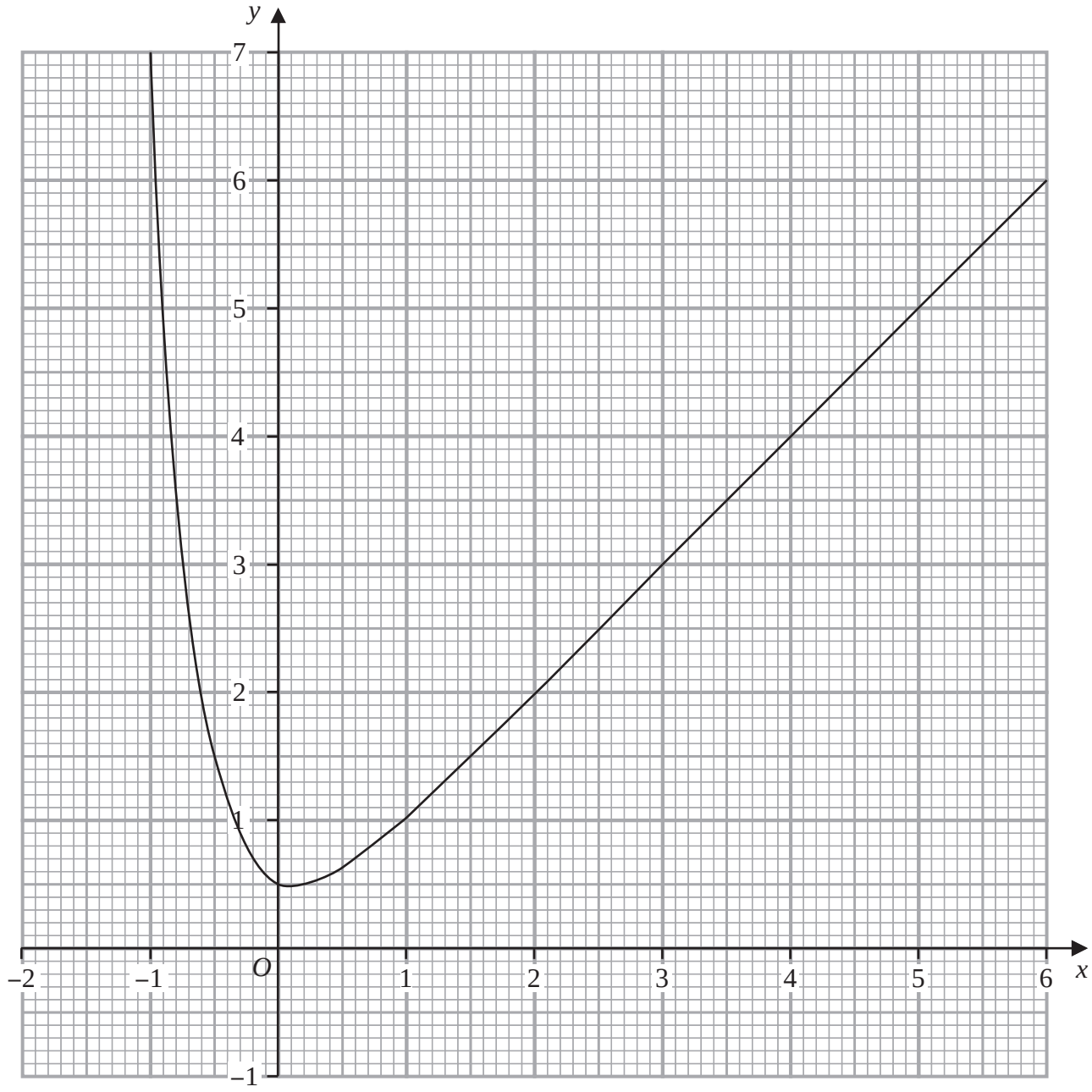

Figure 1

Figure 1 shows part of the graph of the curve with equation $y = x + 2^{-(4x+1)}$

By drawing a suitable straight line on the graph, obtain an estimate, to one decimal place, of the roots of the equation $\log_2(8 - 3x) + 4x = 0$ in the interval $-2 \leqslant x \leqslant 6$

(7)

.....

.....

.....

.....

Question 6 continued

(Total for Question 6 is 7 marks)

7

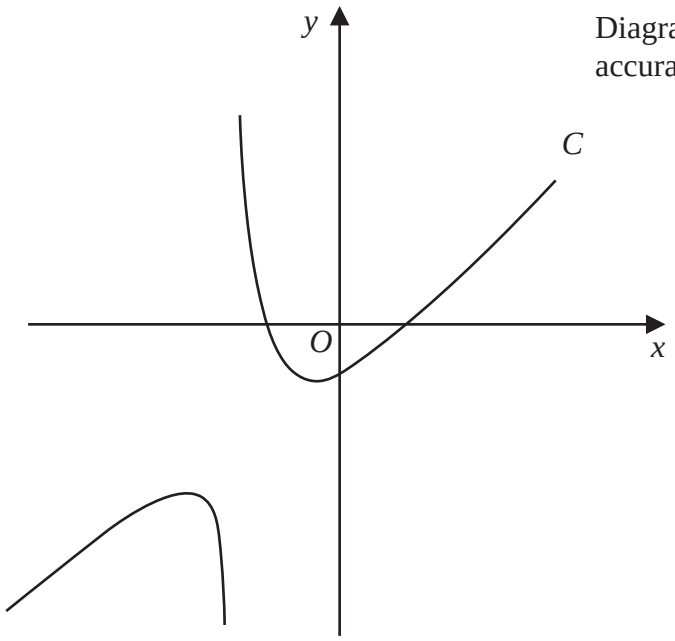

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 - 1}{4x + 5} \quad \text{where} \quad x \neq -\frac{5}{4}$$

(a) Write down the equation of the asymptote to C that is parallel to the y-axis.

(1)

The line l is the normal to C at the point where $x = -1$

(b) Find an equation of l

(7)

The line l meets C again at the point D

(c) Find the coordinates of D

(6)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 14 marks)

8 The sum of the first 2 terms of a geometric series G is 360
The sum of the 2nd and 3rd terms of G is 288

The nth term of G is U_n

(a) Show that $U_n = A\left(\frac{4}{5}\right)^{n-1}$ where A is an integer to be found.

(7)

(b) Explain why G is convergent.

(1)

(c) Hence find the sum to infinity of G

(2)

(d) Find the least number of terms for which the sum is greater than 978

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 14 marks)

9

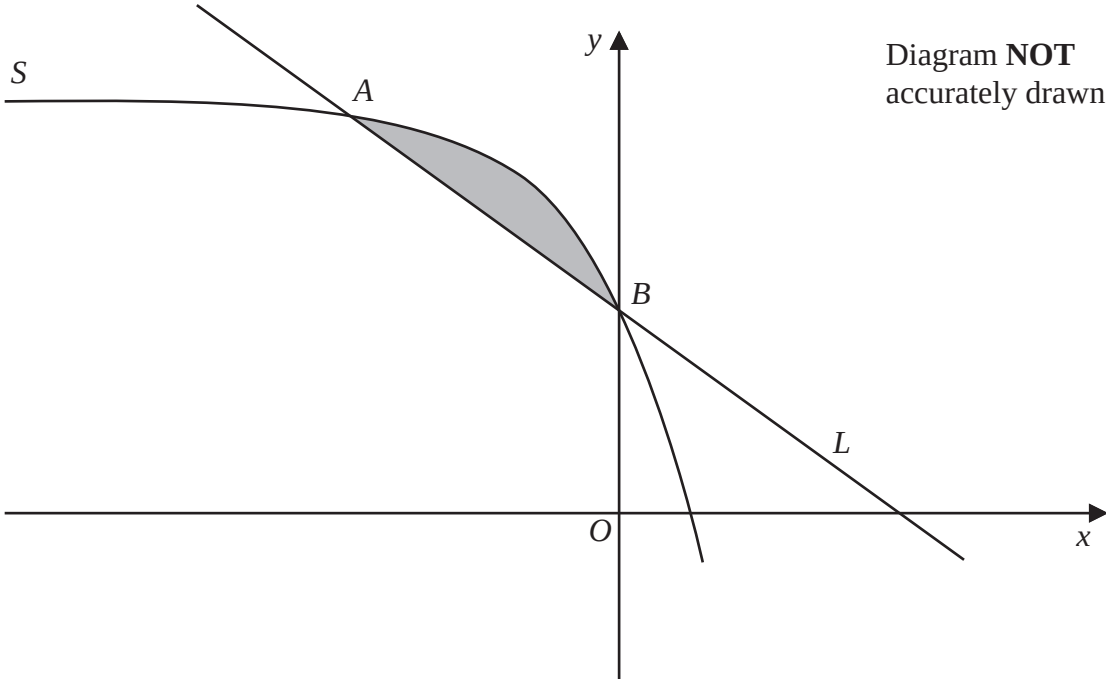

Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a sketch of part of the curve S with equation $y = -2e^{3x} + 4$ and the line L

The curve S has intersections with the line L at the points A and B with x coordinates $x = -1$ and $x = 0$ respectively.

The finite region bounded by S and L is shown shaded in Figure 3

Use calculus to find the exact area of this region.

Give your answer in the form $\dfrac{a + be^{-c}}{c}$ where a, b and c are integers to be found.

(8)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

10

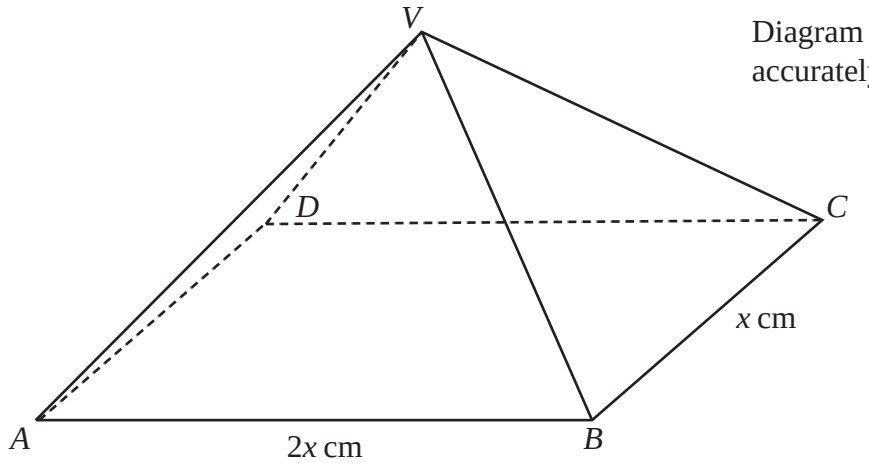

Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows a right pyramid *ABCDV*

The base of the pyramid is a rectangle where,

$$AB = DC = 2x \text{ cm} \quad AD = BC = x \text{ cm}$$

The edges VA, VB, VC and VD are all of equal length.

The angle between *VA* and *ABCD* is 45°

(a) Show that $VA = \frac{\sqrt{10}}{2}x$ cm

(3)

(b) Find in cm, the exact height of the pyramid in terms of *x*

(2)

Find, in degrees to one decimal place,

(c) the size of angle *VBA*

(2)

(d) the size of the obtuse angle between the plane *AVC* and the plane *BVD*

(4)

Given that the volume of the pyramid is $9\sqrt{5}$ cm^3

(e) find the value of *x*

(2)

.....

.....

.....

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 13 marks)

11 (a) Using a formula on page 2 show that $\cos 2A = 2\cos^2 A - 1$

(2)

(b) Hence show that $\left(2\cos^2 A - 1\right)^2 = \dfrac{\cos 4A + 1}{2}$

(3)

The curve with equation $y = \dfrac{\sin 2x}{2} + \dfrac{\left(2\cos^2 x - 1\right)^2}{2} + \dfrac{1}{8}$ has a stationary point P in the range $0 \leqslant x \leqslant \dfrac{\pi}{6}$

(c) Find the exact coordinates of P

(7)

Question 11 continued

Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS